MA106 – Linear Algebra

Assignment 2 **January 2017**

Answer the questions on your own paper. Write your own name in the top left-hand corner, and your supervisor's name in the top right-hand corner. Solutions to Problems **1, 3, 4** *and* **6** *only must be handed in by* **3.00 pm** *on* **MONDAY 30 JANUARY** *(Monday of the fourth week of term), or they will not be marked.*

1. Determine whether the following sequences of vectors are linearly dependent or linearly independent. The field of scalars is $\mathbb{R}$ in each case, and V is $\mathbb{R}^n$ for the appropriate value of n.

(i) $(0,0,1,0)$. [1 mark]

(ii) $(1,-1,0)$, $(1,0,-1)$, $(1,2,-3)$. [2 marks]

(iii) $(0,1,-2)$, $(1,2,3)$, $(3,1,-10)$. [2 marks]

(iv) $(1,0,1,-1)$, $(-1,1,2,2)$, $(0,-2,1,2)$, $(0,1,3,1)$. [2 marks]

2. Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ be vectors in a vector space, and let $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m$ be linear combinations of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$. Suppose that $\mathbf{v}$ is a linear combination of $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m$. Prove that $\mathbf{v}$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$.

3. Determine whether the following sequences of vectors span $\mathbb{R}^n$. (*Hint*: One approach is to determine whether the standard basis vectors $(1,0,\ldots,0), (0,1,0,\ldots,0)$, etc. are linear combinations of the given vectors. If so, then use Qn. 2.)

(i) $(2,1)$, $(-6,-2)$; [2 marks]

(ii) $(2,5,-2)$, $(1,0,-2)$, $(7,3,-4)$, $(-2,1,0)$. [3 marks]

4. Find a basis of $\mathbb{R}^5$ that contains the vectors $(1,-1,1,0,-1)$, $(-1,-1,1,0,-1)$, $(-1,1,1,0,-1)$.

[3 marks]

5. The following sequences of vectors in $\mathbb{R}^4$ are *not* bases of $\mathbb{R}^4$. In each case give a reason why it is not a basis in one short sentence.

(i) $(1,0,1,0)$, $(4,3,2,1)$, $(0,0,0,0)$, $(-2,-1.5,-11,10^{100})$;

(ii) $(1,-1,\sqrt{3},5)$, $(5,0,2,3)$, $(0,3,-2,0)$, $(1,0,1,4)$, $(-4,0,0.5,\pi)$;

(iii) $(6,-3,-2,5)$, $(1,2,1,0)$, $(-1,-2,-1,0)$, $(6,3,2,-5)$;

(iv) $(1,0,1,2)$, $(3,0,-1,-5)$, $(-4,0,-2,6)$, $(1,0,-5,7)$;

(v) $(2,1,-1,2^{0.01})$, $(0,0,0,-2.5)$, $(5,5,5,5)$.

6. Find the dimension and a basis of the following vector spaces V over the given field K. [1 mark for each part]

(i) V is the set of all vectors (α,β,γ) in $\mathbb{R}^3$ with $\alpha+\beta-\gamma=0$; $K=\mathbb{R}$.

(ii) V is the set of all vectors (α,β,γ) in $\mathbb{R}^3$ with $\alpha+\beta=-\gamma$ and $\alpha+2\beta=\gamma$; $K=\mathbb{R}$.

(iii) $V=\mathbb{C}^2$; $K=\mathbb{R}$.

(iv) V is the set of all polynomials over $\mathbb{R}$ of degree at most n, for which the third derivative vanishes.

(v) $K=\mathbb{R}$ and V is the set of functions from $\mathbb{R}$ to $\mathbb{R}$ which are solutions of the differential equation $\frac{d^2 f}{dx^2}+2f=0$.